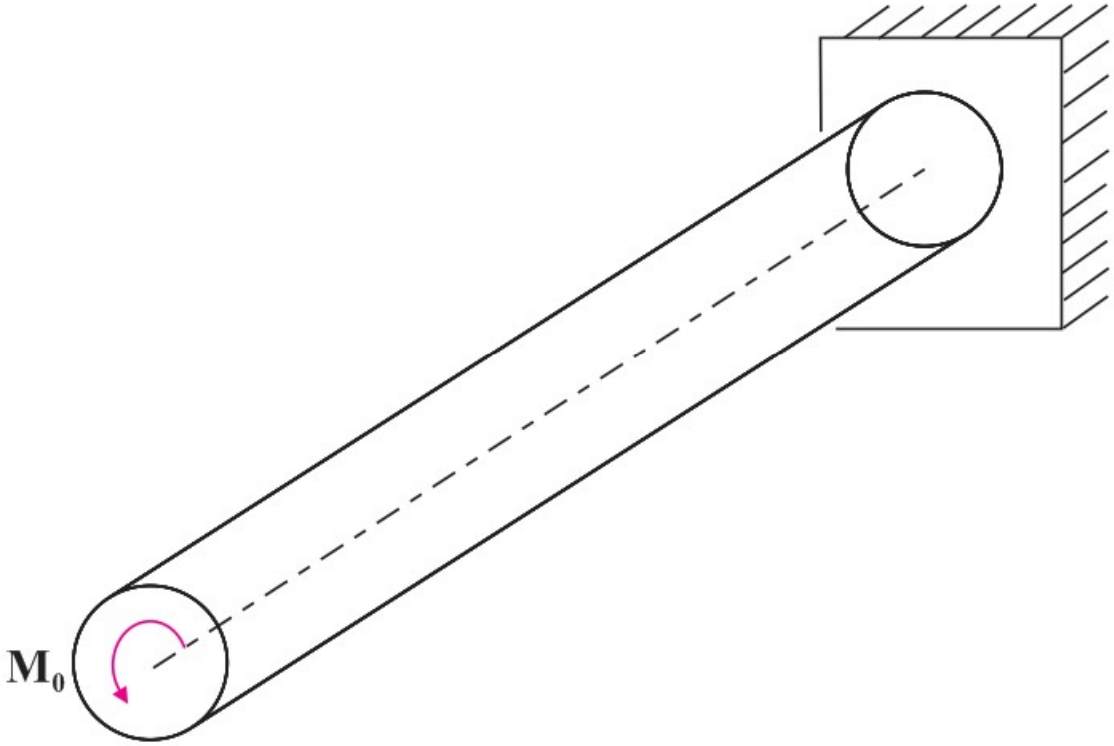
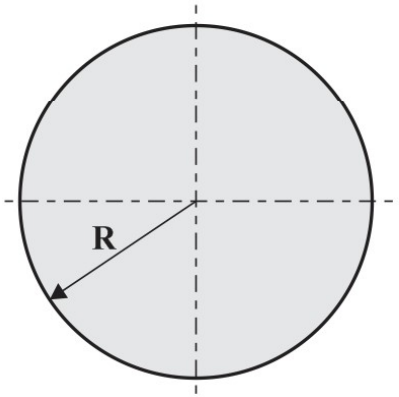
UVIJANJE (Slobodno)

SLOBODNO UVIJANJE ŠTAPOVA KRUŽNOG POPREČNOG PRESEKA

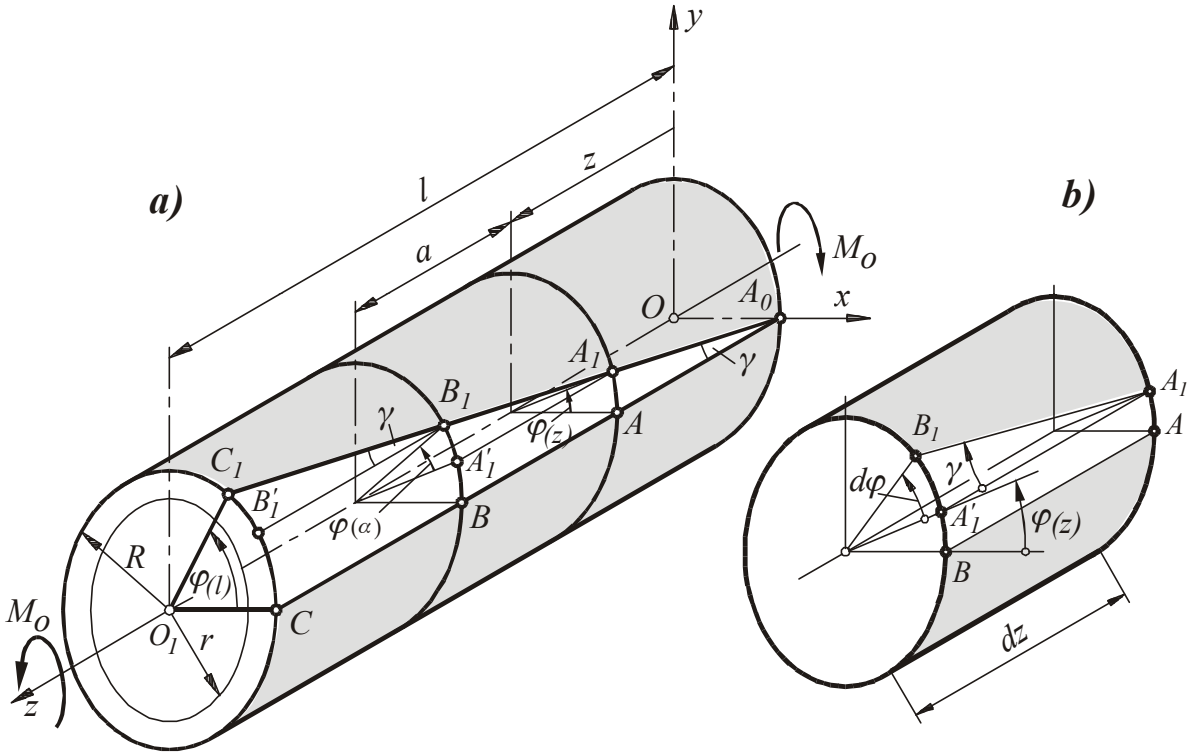




Sen Venan (1797-1886)

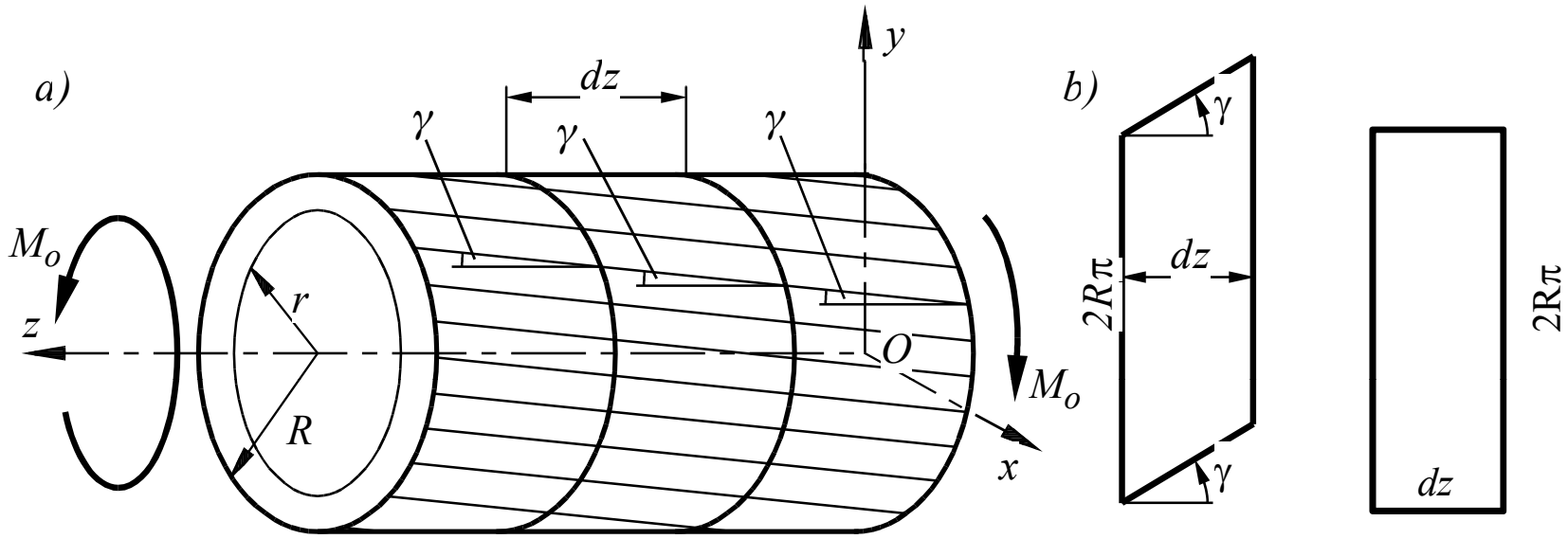
Eksperiment >

pri uvijanju štapa kružnog poprečog preseka poprečni preseci ostaju ravni (nema deplanacije)



Slobodno (čisto) uvijanje

- ne postoje normalni naponi (nema deformacije u pravcu ose z)



Deformacija: KLIZANJE

- nagib zamišljenih izvodica za ugao γ

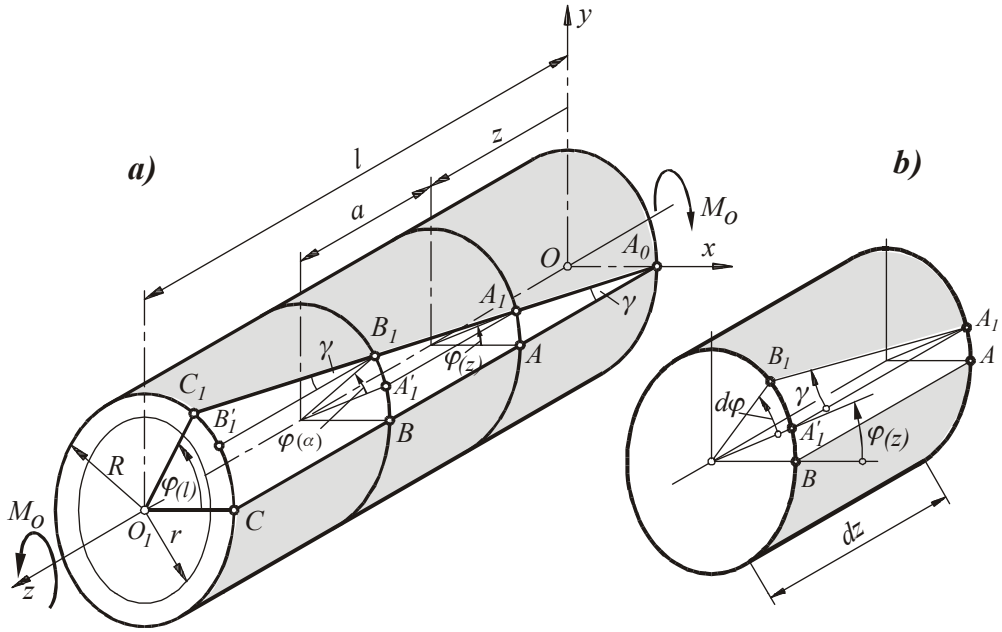
$$tg\gamma(r) \cong \gamma(r) = \frac{\varphi_{(l)} \cdot r}{l} = \theta \cdot r$$

Ugao uvijanja po jedinici dužine štapa

$$\theta = \frac{\varphi_{(l)}}{l}$$

Ugao uvijanja

$$\varphi(z) = \theta \cdot z$$



Tangencijali napon (po Hukovom zakonu)

$$\tau(r) = G \cdot \gamma(r) = G \cdot \theta \cdot r$$

$$\theta = \frac{\varphi_{(l)}}{l}$$

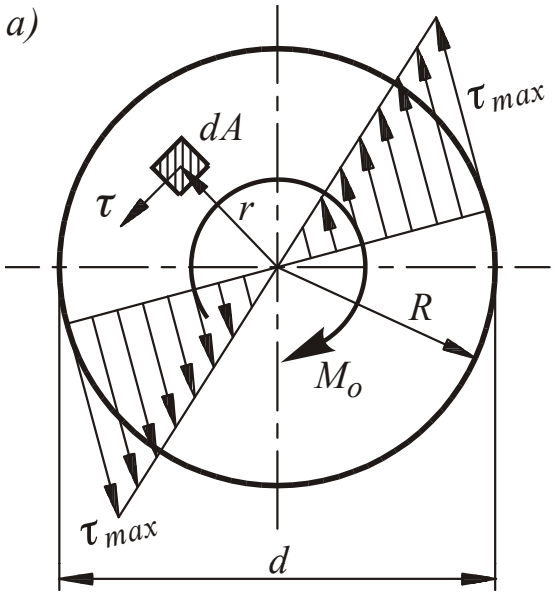
Jednačina ravnoteže

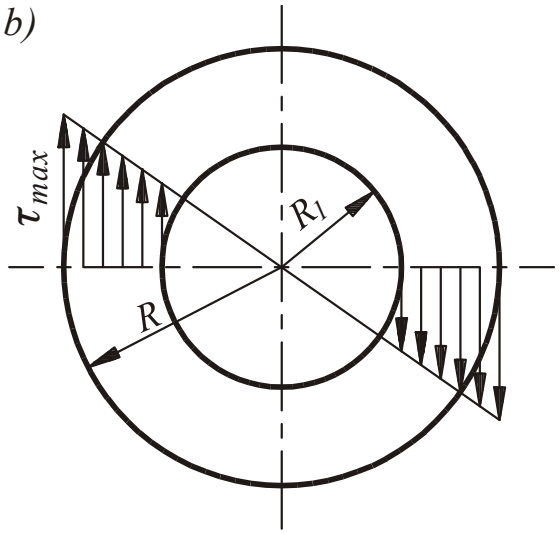
$$M_o = M_\tau = \int_A \tau \cdot r \cdot dA = \theta \cdot G \int_A r^2 dA = \theta \cdot G \cdot I_0$$

Polarni momet inercije

$$\theta = \frac{M_o}{G I_0}$$

$$\varphi(z) = \theta \cdot z \qquad \varphi(z) = \theta \cdot z = \frac{M_o \cdot z}{G I_0} \qquad \tau(r) = \frac{M_0}{I_0} \cdot r$$





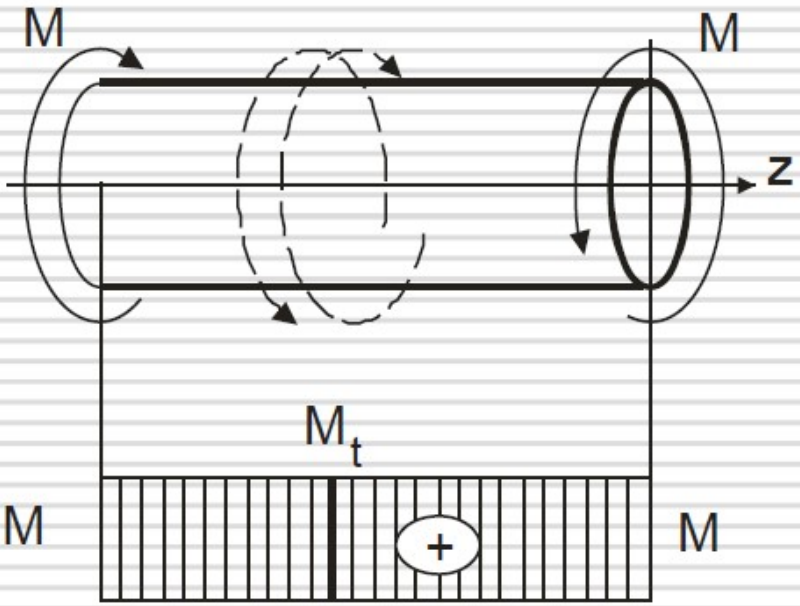
$$\tau(r) = \frac{M_0}{I_0} \cdot r$$

$$I_0 = \int_0^R 2\pi r^3 dr = \frac{\pi R^4}{2} = \frac{\pi d^4}{32} \qquad (dA = 2\pi r\, dr)$$

$$I_{01} = \frac{\pi}{2} \cdot \left(R^4 - R_1^4\right)$$

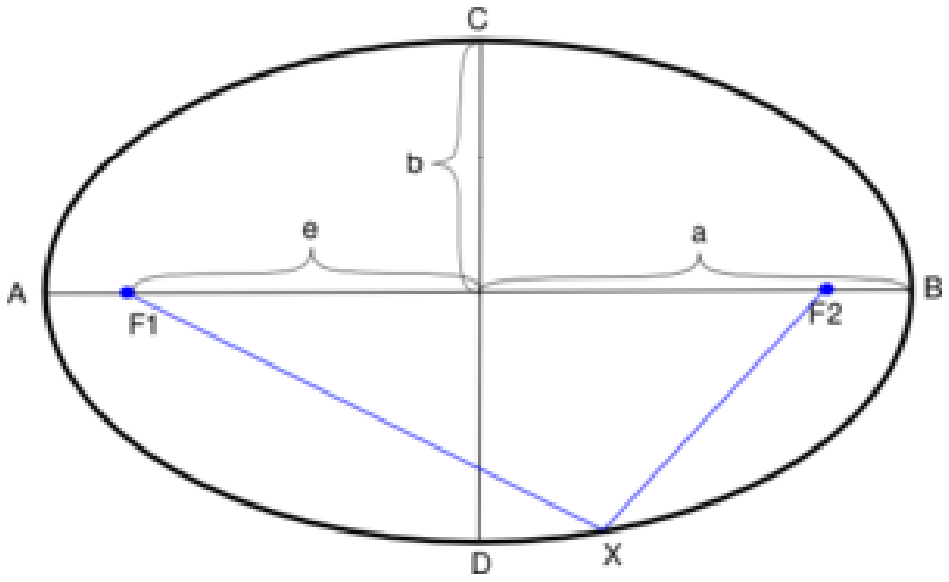
$$\tau_{max} = \frac{M_0}{I_0} \cdot R = \frac{16 M_0}{\pi d^3} \cong \frac{M_0}{0{,}2 \cdot d^3}$$

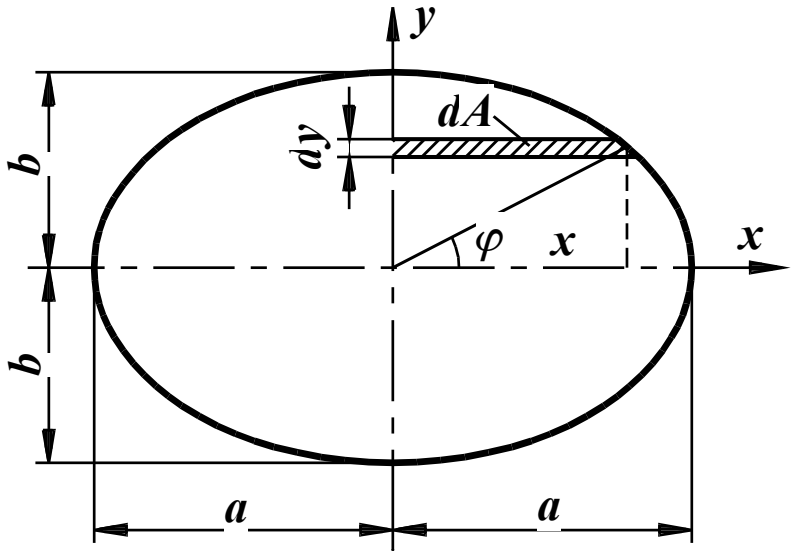
$$\tau_{max} = \frac{M_0}{I_{01}} \cdot R, \quad \varphi(l) = \frac{M_0 \cdot l}{G I_{01}}$$



- Moment uvijanja M_t, unutrašnji moment, smatra se pozitivnim ako obrće u smeru kazaljke na časovniku posmatran iz vrha normale na ravan momenta
- Dijagram momenta uvijanja analiziranog štapa

Slobodno uvijanje štapa eliptičnog poprečnog preseka





$$1-\frac{x^2}{a^2}-\frac{y^2}{b^2}=0$$

$$A=\iint dx\,dy=4\int_A dA=4\int x dy=\;:ab\pi$$

$$I_x=\iint y^2\,dx\,dy=4\int_0^{\frac{\pi}{2}} y^2\,x\,dy \qquad I_x=\frac{1}{4}a\,b^3\pi$$

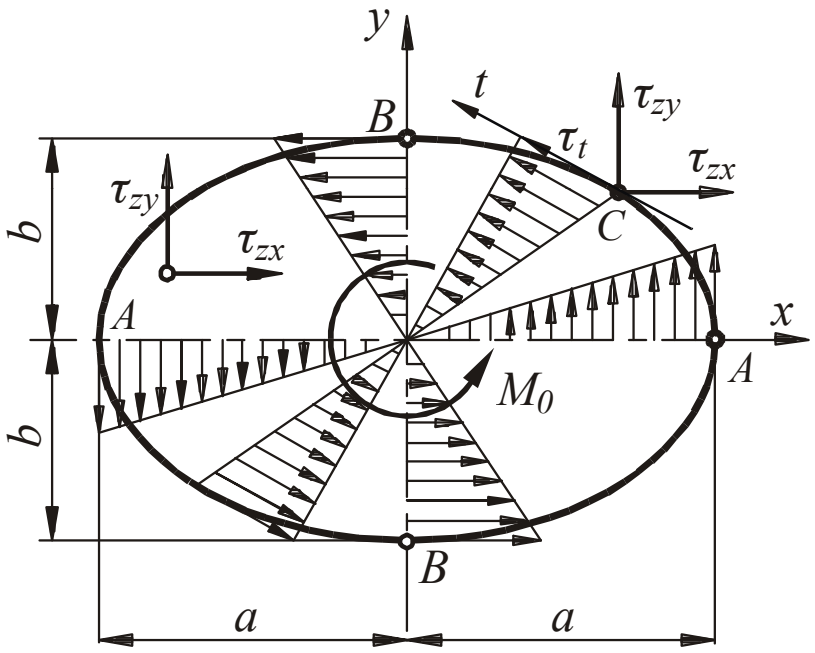
$$I_y=\iint x^2\,dx\,dy=\frac{1}{4}a^3 b\pi$$

$$M_0=\quad G\theta\pi\,\frac{a^3b^3}{a^2+b^2}$$

Torzioni moment inercije

$$I_t = \frac{M_0}{G\theta} = \pi \frac{a^3 b^3}{a^2 + b^2}$$

Tangencijalni napon $$\tau_t = \sqrt{\tau_{zx}^2 + \tau_{zy}^2} = \frac{2M_0}{\pi a^2 b^2} \sqrt{\left(\frac{bx}{a}\right)^2 + \left(\frac{ay}{b}\right)^2}$$



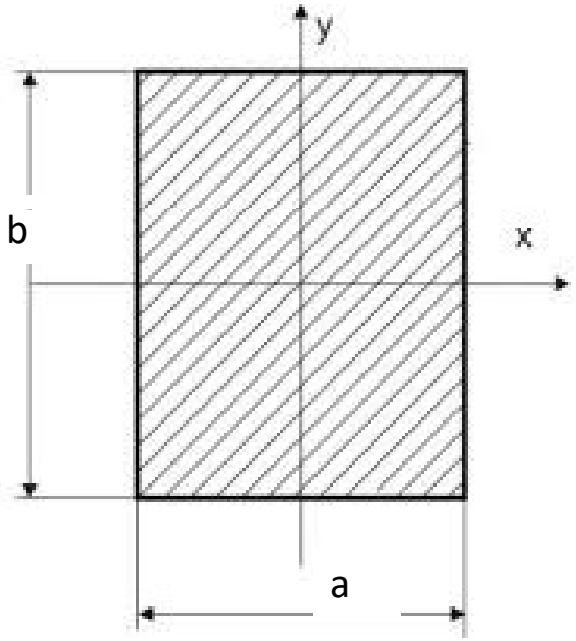
U ta~kama $A(a,0)$ i $B(0,b)$ naponi iznose:

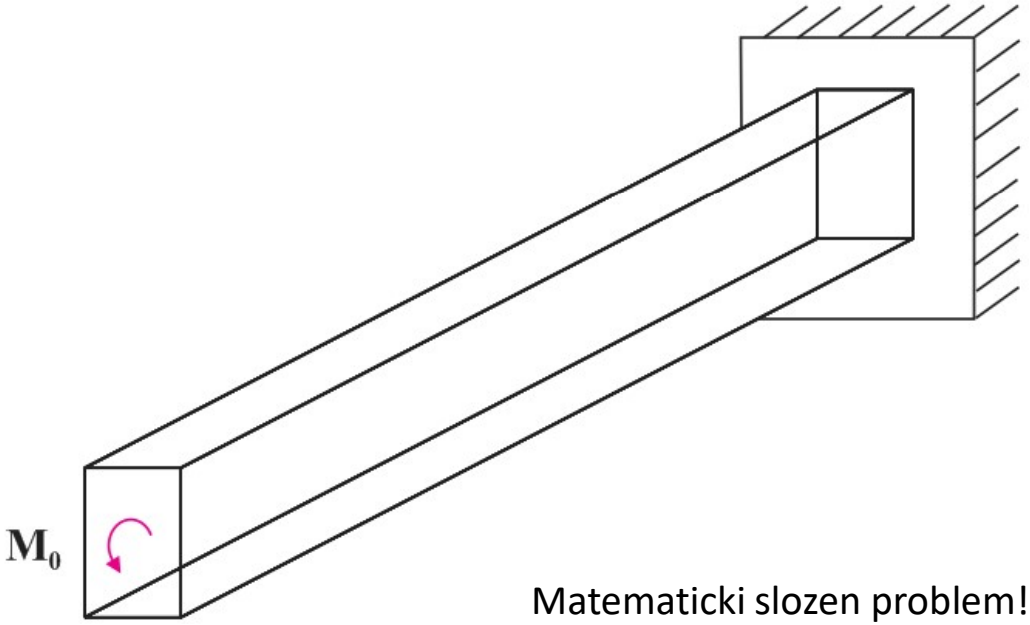
$$\tau_{zxA} = 0, \quad \tau_{zyA} = 2G\theta \frac{ab^2}{a^2 + b^2},$$

$$\tau_{zxB} = -2G\theta \frac{a^2 b}{a^2 + b^2}, \quad \tau_{zyB} = 0.$$

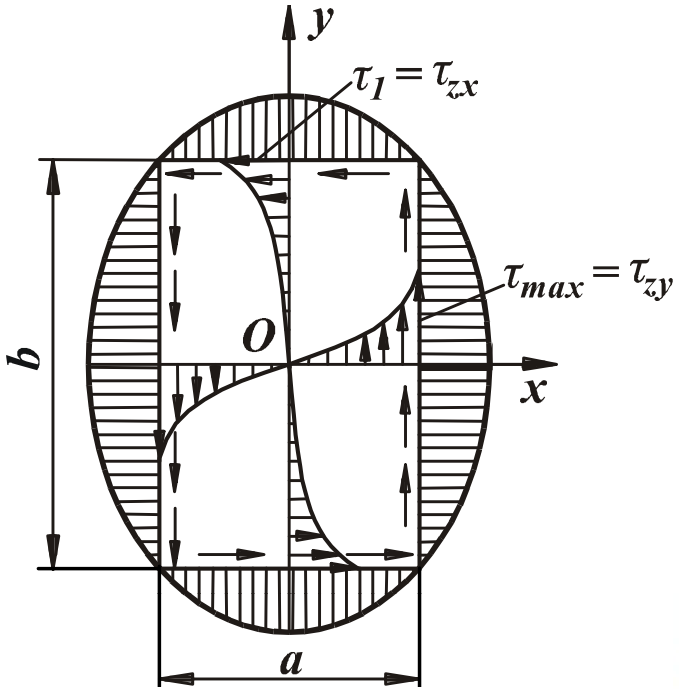
Zaključak: ?

Slobodno uvijanje – pravougaoni poprečni presek





Matematicki slozen problem!



$$M_0 = G\theta \frac{a^3 b}{3}\left[1 - \frac{192a}{\pi^5 b}\sum_{n=1,3,5,}^{\infty}\frac{1}{n^5}\,th\frac{n\pi b}{2a}\right].$$

$$I_t = \frac{M_0}{G\theta} = \frac{a^3 b}{3}\left[1 - \frac{192a}{\pi^5 b}\sum_{n=1,3,5,}^{\infty}\frac{1}{n^5}\,th\frac{n\pi b}{2a}\right]$$

$$I_t = k_1 a^3 b,$$

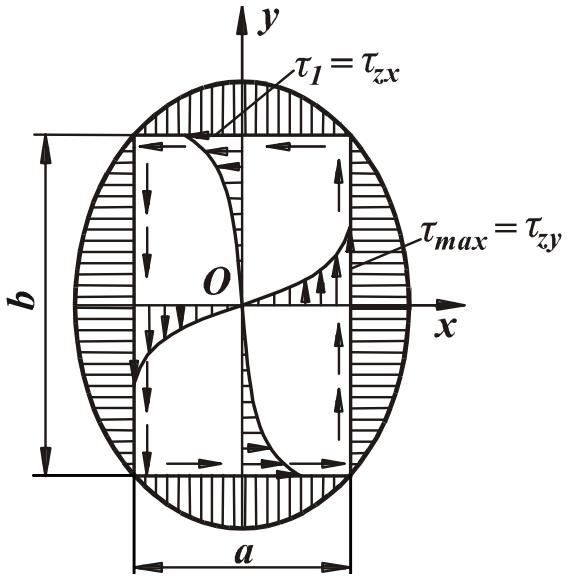
$$M_0 = I_t G\theta = k_1 a^3 b G\theta,$$

$$\tau = \frac{M_0}{W_t}$$

$$W_t = \frac{k_1 a^2 b}{\frac{8}{\pi^2} \sum_{n=1,3,5,\ldots}^{\infty} \frac{1}{n^2} \left(1 - \frac{1}{ch \frac{n\pi b}{2a}}\right)} = k_2 a^2 b,$$

$$k_2 = \frac{k_1}{\frac{8}{\pi^2} \sum_{n=1,3,5,..}^{\infty} \frac{1}{n^2} \left(1 - \frac{1}{ch \frac{n\pi b}{2a}}\right)}$$

Uvojni otporni moment



$$\tau_1 = k_3 \ \tau_{max}$$

b/a	1	1,5	2	2,5	3	4	10	∞
k_1	0,1406	0,196	0.229	0,249	0,263	0,281	0,312	0,333
k_2	0,208	0,231	0,246	0,258	0,267	0,282	0,312	0,333
k_3	1,0	0,859	0,795	0,766	0,753	0,745	0,742	0,742

$$I_t = k_1 a^3 b$$